ANOTHER THEORY OF GRAVITATION

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ABSTRACT

For four centuries, Galileo and Eötvös have shown in several experiments the equality of the gravitational mass and the inertial mass. Einstein has explained this equality by the equivalence principle (EQ), but other possibilities exist.

The reason of the equality of the gravitational mass and the inertial mass could be much deeper hence the introduction of the EQ is not necessary.

To show it, we consider a test particle P of inertial mass m and gravitational mass M in a Newtonian gravitational field U. If the ratio K=M/m is variable the mass M is a function of U.

Consequently the internal energy of the particle E=Mc² depends on U. However a variation of E implies a correlated variation of the fundamental constants.

But the existence of a stable universe (allowing the construction of complex and durable systems) claims, on the contrary, that these quantities remain constant. Therefore it is not need to resort to the principle of equivalence.

The theory developed in this paper makes the distinction between inertial field and gravitational field but with K=1. As the RG it explains the advance of the Mercury perihelion, the deviation of the light by the Sun and the Mössbauer effect.

But contrary to the RG this theory authorizes the existence of stable celestial bodies whose mass is not limited.

The metric one thus obtained does not present any singularity (in RG, Schwarzschild singularity). In this theory the concept of black hole disappears.

CONVENTIONS AND ABBREVIATIONS

The sign conventions for the metric and curvature tensors are (-, +, +) in the terminology of Mismer, Thorne & Wheeler [1]. That is, the metric signature is (+, -, -, -).

For this paper we use geometric units in which c = G =1. (Except the § 4.22)

The following symbols and abbreviations are used throughout:

∂_μ or $_{,\mu}$	partial derivative
D_μ or $_{;\mu}$	covariant derivative
ln	natural logarithm
i, j, k,….	Latin indices equal to 1, 2 & 3
λ, μ, ν,…	Greek indices equal to 0, 1, 2, & 3
cst	constant quantity

$\Box$	laplacian on a four dimensional manifold
$[\ ,\ ]_L$	Lie's brackets
$\approx$	Asymptotically equal to
#	Approximately equal to

1 – INTRODUCTION AND HYPOTHESIS [5].

The study of the movement of the non charged matter lead to consider that the space-time is a four dimensional differentiable Riemannian manifold U whose the metric tensor g has the signature (+, -, -, -).

$$ds^2 = g_{\lambda\mu}\, dx^\lambda\, dx^\mu \tag{1.1}$$

For example the space-time of the rotating disk is not flat.

On the other hand we reject, with J. L. Synge, the weak equivalence principle. We utilise, for describe the non charged matter, three fields on the manifold U:

- The inertial field who is a field of symmetric connection Γ.
- The matter field who is a field q taking its values in a three dimensional manifold.
- The gravitational field who is a real scalar field Φ.

From now we suppose that the matter is a perfect fluid with an equation of state $\rho = \varphi(p)$ where ρ is the density and p the pressure of the fluid, the habitual hypothesis of approximation lead to the lagrangians :

$$L_{inert} = g^{\lambda\mu} R_{\lambda\mu} \sqrt{(-g)}$$
$$L_{grav} = 2\Phi_{,\lambda}\, \Phi^{,\lambda} \sqrt{(-g)}$$
$$L_{mat} = 16\pi\rho \sqrt{(-g)}$$
$$L_{mat + grav} = 16\pi\rho\, f(\Phi)\sqrt{(-g)}$$
$$L = L_{inert} + L_{grav} + L_{mat + grav}$$
$$L = \{\, g^{\lambda\mu} R_{\lambda\mu} + 2\Phi_{,\lambda}\, \Phi^{,\lambda} + 16\pi\rho\, f(\Phi)\, \} \sqrt{(-g)} \tag{1.2}$$

Where:

- $R_{\lambda\mu}$ is the Ricci tensor of the connection Γ.
- $\rho = \rho(q^j, \det(q^j_{,\lambda}\, q^{k,\lambda}))$.
- f is a function describing the interaction between the matter and the gravitational field, with f(0) =1.

The constants in (1.2) are done by choice of the units.

The eulerian equations for Γ show that Γ is the riemannian connection of U [9 p. 338 to 345].

The other equations are (we don't write the equations for the q^j):

$$\Box\Phi = 4\pi f\,'(\Phi)\rho \tag{1.3}$$
$$R_{\lambda\mu} - \tfrac{1}{2} R\, g_{\lambda\mu} = 8\pi\, T_{\lambda\mu} \tag{1.4}$$

Where $R = g^{\lambda\mu} R_{\lambda\mu}$ is the Riemannian curvature of U, $\Box\Phi = \Phi^{,\lambda}{}_{;\lambda}$ and :

$T_{\lambda\mu} = T_{\lambda\mu}(L_{mat + grav}) + T_{\lambda\mu}(L_{grav})$

But $f(\Phi)$ is independent of $g_{\lambda\mu}$ hence we have :

$T_{\lambda\mu}(L_{mat + grav}) = f(\Phi)\, T_{\lambda\mu}(L_{mat})$

$$T_{\lambda\mu} = f(\Phi)((\rho +p)u_{\lambda}u_{\mu} - p\, g_{\lambda\mu}) - \{\Phi_{,\lambda}\, \Phi_{,\mu} - ½\, g_{\lambda\mu}\, \Phi_{,\lambda}\, \Phi^{,\lambda} \}/4\pi \qquad (1.5)$$

<u>2 – HOLONOMIC MEDIUMS</u> [2].

If we assume that U contains a material distribution (in interaction with a gravitational field or no) such as the stress-energy tensor can be written:

$$T_{\lambda\mu} = r\, u_{\lambda}\, u_{\mu} - \theta_{\lambda\mu} \qquad (2.1)$$

Where:

r is a positive scalar.
u_{λ} is the 4- velocity of the medium.
$\theta_{\lambda\mu}$ is a symmetrical covariant tensor.

Then the distribution described by $T_{\lambda\mu}$ can be called a **holonomic medium** if and only if the vector K defined by:

$$r\, K_{\mu} = D_{\lambda}\, \theta^{\lambda}{}_{\mu} \qquad (2.2)$$

is a gradient. So we take:

$$K_{\lambda} = \partial_{\lambda} \ln F \qquad (2.3)$$

r being the pseudo-density and F the index of the distribution.
In that case the flow lines of the medium are geodesics of the conformal metric:

$$d\sigma^2 = F^2\, ds^2 = \gamma_{\lambda\mu}\, dx^{\lambda}\, dx^{\mu} \qquad (2.4)$$

The tensor metric γ is thus the only one having physical reality. Consequently, the notions of time and space must be deduced from it.

We define the **vortex tensor** of the medium by:

$$\Omega_{\lambda\mu} = \partial_{\lambda} (Fu_{\mu}) - \partial_{\mu} (Fu_{\lambda}) \qquad (2.5)$$

A. Lichnerowicz says that the motion of a holonomic medium is without vortex or **irrotational** if and only if:

$$\Omega_{\lambda\mu} = 0 \qquad (2.6)$$

It is important to remember that a perfect fluid of density ρ and pressure p has a stress-energy tensor:

$$T_{\lambda\mu} = (\rho + p)\, u_\lambda u_\mu - p\, g_{\lambda\mu} \tag{2.7}$$

If an equation of state **ρ = φ(p)** exits the perfect fluid is a holonomic medium with:

$$r = \rho + p \qquad\qquad F = \exp\left(\int dp /(\rho + p)\right) \tag{2.8}$$

3 – COMOVING COORDINATE SYSTEMS AND ABSOLUTE TIME [3], [4], [6], [7].

Definition. It is said that a coordinate system of U is comoving if and only if:

$$u^i = 0 \tag{3.1}$$

Hence, we have:

$$u^0 = 1/ \surd(g_{00}) \qquad u^\lambda = \delta^\lambda{}_0 / \surd(g_{00}) \qquad u_\lambda = g_{0\lambda} / \surd(g_{00}) \tag{3.2}$$

Theorem 3.1 Let a holonomic medium then it exists a comoving coordinate system such we have:

$$d\sigma^2 = (dx^0)^2 + 2\, \gamma_{0i}\, dx^0 dx^i + \gamma_{ij}\, dx^i dx^j \tag{3.3}$$

with

$$\partial_0 \gamma_{0i} = 0 \tag{3.4}$$

Proof. With the possible coordinate transformations we can choose the value of four quantities, hence it exists a comoving coordinate system such that $\gamma_{00} = 1$ i.e.

$$u^1 = u^2 = u^3 = 0 \ \ \& \ \ \gamma_{00} = 1$$

We note $\Gamma^\lambda{}_{\mu\nu}$ the Christoffel symbol of $d\sigma^2$, the geodesic equation of $d\sigma^2$ is:

$$d^2x^\lambda/d\sigma^2 + \Gamma^\lambda{}_{\mu\nu}\, (dx^\mu/d\sigma)(dx^\nu/d\sigma) = 0 \tag{3.5}$$

The coordinates are comoving, hence the curves (x^1, x^2, x^3) =cst are geodesic i.e.

$$dx^\mu/d\sigma = \delta^\mu{}_0$$

(3.5) gives $\Gamma^\lambda{}_{00} = 0$ hence

$$\Gamma^i{}_{00} = ½\, \gamma^{i\lambda}\, (\partial_0 \gamma_{0\lambda} + \partial_0 \gamma_{\lambda 0} - \partial_\lambda \gamma_{00}) = 0$$

Hence

$$\gamma^{ij}\, \partial_0 \gamma_{0j} = 0$$

And

$\partial_0 \gamma_{0i} = 0$

That completes the proof.

Theorem 3.2 Let a holonomic medium where the motion is without vortex:

1) It exists a comoving coordinate system such that:

$$d\sigma^2 = dt^2 - \eta_{ij}\, dx^i dx^j \tag{3.6}$$

$$ds^2 = dt^2 / F^2 - h_{ij}\, dx^i dx^j \tag{3.7}$$

Where h_{ij} is definite positive.

2) $r \sqrt{(h)} / F = C(x^1, x^2, x^3)$ (3.8)

Where $h = \det(h_{ij})$.

Proof.

Firstly, we apply the theorem 1 and we utilize a comoving coordinate system satisfying to (3.3) & (3.4).

$F^2 g_{00} = \gamma_{00} = 1$

$g_{00} = 1/ F^2$

We consider the vorticity tensor:

$\Omega_{\lambda\mu} = \partial_\lambda (F u_\mu) - \partial_\mu (F u_\lambda)$

$\Omega_{\lambda\mu} = \partial_\lambda (F^2 g_{0\mu}) - \partial_\mu (F^2 g_{0\lambda})$

$\Omega_{\lambda\mu} = \partial_\lambda \gamma_{0\mu} - \partial_\mu \gamma_{0\lambda}$

The movement is without vortex hence:

$\Omega_{\lambda\mu} = 0$

Hence with (3.4)

$\partial_i \gamma_{0j} = \partial_j \gamma_{0i} \qquad \partial_0 \gamma_{0i} = 0$

Hence it exits a numerical function $C = C(x^1, x^2, x^3)$ such as:

$\gamma_{0i} = \partial_i f$

Let $t = x^0 + f$

$dt = dx^0 + \partial_i f\, dx^i = dx^0 + \gamma_{0i}\, dx^i$

$dt^2 = (dx^0)^2 + 2\, \gamma_{0i}\, dx^0 dx^i + \gamma_{0i}\, \gamma_{0j}\, dx^i dx^j$

$(dx^0)^2 + 2\, \gamma_{0i}\, dx^0 dx^i = d\tau^2 - \gamma_{0i}\, \gamma_{0j}\, dx^i dx^j$

We put in (3.3)

$d\sigma^2 = dt^2 + (\gamma_{ij} - \gamma_{0i}\, \gamma_{0j})dx^i dx^j$

Let $\eta_{ij} = \gamma_{0i}\, \gamma_{0j} - \gamma_{ij}$

We obtain (3.6)

$d\sigma^2 = dt^2 - \eta_{ij}\, dx^i dx^j$

Lastly with $h_{ij} = \eta_{ij} / F^2$ we are:

$ds^2 = d\sigma^2 / F^2 = d\tau^2 / F^2 - h_{ij}\, dx^i dx^j$

Secondly, we write the conservation identities.

$D_\lambda\, T^\lambda{}_\mu = 0$

$D_\lambda\, (r\, u^\lambda\, u_\mu) - D_\lambda\, \theta^\lambda{}_\mu = 0$

$D_\lambda\, (r\, u^\lambda\, u_\mu) - r\, \partial_\lambda F / F = 0$

We use a classical expression of the divergence of a symmetric tensor and the components of the 4-velocity.

$u^\lambda = F\, \delta^\lambda{}_0$ & $u_\lambda = \delta^0{}_\lambda / F$

$\partial_\lambda\, (r\, \delta^\lambda{}_0\, \delta^0{}_\mu \sqrt{(-g)}\,) / \sqrt{(-g)} - ½\, (\partial_\mu\, g_{\alpha\beta})\, (r\, \delta^\alpha{}_0\, \delta^\beta{}_0\, F^2) - r\, \partial_\mu F / F = 0$

Where $g = \det(g_{\lambda\mu}) = h / F^2$

Therefore

$\partial_\lambda\, (r\, \delta^\lambda{}_0\, \delta^0{}_\mu \sqrt{(h)} / F\,)F / \sqrt{(h)} - ½\, (\partial_\mu\, g_{00})\, (r\, F^2) - r\, \partial_\mu F / F = 0$

But $g_{00} = 1/ F^2$

$\partial_0\, (r\, \delta^0{}_\mu \sqrt{(h)} / F\,)F / \sqrt{(h)} - ½\, (-2\, \partial_\mu F / F^3)\, (r\, F^2) - r\, \partial_\mu F / F = 0$

∂_0 (r $\delta^0{}_\mu$ $\sqrt{(h)}$ / F)F / $\sqrt{(h)}$ = 0

∂_0 (r $\delta^0{}_\mu$ $\sqrt{(h)}$ / F) = 0

∂_0 (r $\sqrt{(h)}$ / F) = 0

That completes the proof.

The two theorems preceding have an important consequence.

The time t is the same for all points of U in relative rest. Therefore this is an absolute time defined with a univocal manner.

4 – FUNDAMENTAL PROPERTIES OF THE GRAVITATIONAL FIELD

4.1 – TRAJECTORIES IN A GRAVITATIONAL FIELD

We consider a gravitational field interacting with a perfect fluid; we have with the notations of the paragraph 1:

$$T_{\lambda\mu} = f(\Phi)((\rho + p)u_\lambda u_\mu - p\, g_{\lambda\mu}) - \{\Phi_{,\lambda}\,\Phi_{,\mu} - \tfrac{1}{2} g_{\lambda\mu}\,\Phi_{,\lambda}\,\Phi^{,\lambda}\}/4\pi \qquad (4.1)$$

Theorem 4.1 A gravitational field interacting with a perfect fluid is a holonomic medium with a pseudo-density:

$$r = (\rho + p)\, f(\Phi) \qquad (4.2)$$

and an index :

$$F = f(\Phi)F_0 \qquad (4.3)$$

where $F_0 = \exp(\int dp/(\rho + p))$ is the index of the fluid only.

More over the trajectory of a test-body in a gravitational field is a geodesic of the conformal metric :

$$d\sigma^2 = (f(\Phi)F_0)^2\, ds^2 \qquad (4.4)$$

Proof.

Necessary we have r = (ρ + p) f(Φ) and :

$T_{\lambda\mu} = f(\Phi)((\rho + p)u_\lambda u_\mu - p\, g_{\lambda\mu}) - \{\Phi_{,\lambda}\,\Phi_{,\mu} - \tfrac{1}{2} g_{\lambda\mu}\,\Phi_{,\lambda}\,\Phi^{,\lambda}\}/4\pi$

$T^\lambda{}_\mu = f(\Phi)((\rho + p)u^\lambda u_\mu - p\, g^\lambda{}_\mu) - \{\Phi^{,\lambda}\,\Phi_{,\mu} - \tfrac{1}{2} g^\lambda{}_\mu\,\Phi_{,\lambda}\,\Phi^{,\lambda}\}/4\pi$

$T^\lambda{}_\mu = r\, u^\lambda u_\mu - \theta^\lambda{}_\mu$

Where :

$\theta^\lambda{}_\mu = f(\Phi)p g^\lambda{}_\mu + \{\Phi^{,\lambda}\,\Phi_{,\mu} - \tfrac{1}{2} g^\lambda{}_\mu\,\Phi_{,\lambda}\,\Phi^{,\lambda}\}/4\pi$

$D_\lambda\theta^\lambda{}_\mu = \partial_\mu(f(\Phi)p) + \{D_\lambda(\partial^\lambda\Phi)\,\partial_\mu\Phi + \partial^\lambda\Phi\, D_\lambda(\partial_\mu\Phi) - D_\mu(\partial_\lambda\Phi)\,\partial^\lambda\Phi\}/4\pi$

$D_\lambda\theta^\lambda{}_\mu = \partial_\mu(f(\Phi)p) + \{D_\lambda(\partial^\lambda\Phi)\,\partial_\mu\Phi + \partial^\lambda\Phi\,[D_\lambda(\partial_\mu\Phi) - D_\mu(\partial_\lambda\Phi)]\}/4\pi$
$D_\lambda\theta^\lambda{}_\mu = \partial_\mu(f(\Phi)p) + \{D_\lambda(\partial^\lambda\Phi)\,\partial_\mu\Phi + \partial^\lambda\Phi\,[\partial_\lambda, \partial_\mu]_L\Phi\}/4\pi$
$D_\lambda\theta^\lambda{}_\mu = \partial_\mu(f(\Phi)p) + \Box\Phi\,\partial_\mu\Phi/4\pi$
$D_\lambda\theta^\lambda{}_\mu = \partial_\mu(f(\Phi)p) + f'(\Phi)\rho\,\partial_\mu\Phi$
$D_\lambda\theta^\lambda{}_\mu = (\rho + p)\,f'(\Phi)\partial_\mu\Phi + f(\Phi)\partial_\mu p$
$D_\lambda\theta^\lambda{}_\mu = (\rho + p)\,f(\Phi)[f'(\Phi)\partial_\mu\Phi/f(\Phi) + \partial_\mu p/(\rho + p)]$
$D_\lambda\theta^\lambda{}_\mu = (\rho + p)\,f(\Phi)[\partial_\mu \ln f(\Phi) + \partial_\mu \ln F_0]$
$D_\lambda\theta^\lambda{}_\mu = r\,\partial_\mu \ln (f(\Phi)F_0)$

Hence, by definition, the medium is holonomic and, by virtue of the paragraph 2, the trajectory of a test-body in a gravitational field is a geodesic of the conformal metric:

$$d\sigma^2 = (f(\Phi)F_0)^2\, ds^2$$

For the determination of the function f see § 4.31.

4.2 – THE GRAVITATIONAL FIELD IN VACUUM

4.21 –EQUATIONS WITH SPHERICAL SYMMETRY

In vacuum we have $\rho = p = 0$ and the trajectory of a test- body is a geodesic of the metric ds^2.

We write the metric ds^2 with a spherical symmetry:

$$ds^2 = e^{2a}dt^2 - e^{2b}\,(dr^2 + r^2\,(d\theta^2 + \sin^2\theta\, d\varphi^2)) \qquad (4.5)$$

where a and b are some functions of r.
We have $\rho = 0$ and Φ is a function of r, the Einstein's equations give :

$$4\,b' + r\,b'^2 - r\,\Phi'^2 + 2\,r\,b'' = 0 \qquad (4.6)$$

$$2\,a' + 2\,b' + 2\,r\,a'\,b' + r\,b'^2 + r\,\Phi'^2 = 0 \qquad (4.7)$$

$$a' + r\,a'^2 + b' - r\,\Phi'^2 + r\,a'' + r\,b'' = 0 \qquad (4.8)$$

We can add the field equation for Φ :

$$(2/r + a' + b')\,\Phi' + \Phi'' = 0 \qquad (4.9)$$

The complete integration of these equations is easy, but we have a particular important solution:

$$b = - a = \Phi = m / r \qquad (4.10)$$

We see that Φ is similar to the Newtonian potential and we have:

$$ds^2 = e^{-2\Phi}dt^2 - e^{2\Phi}\,(dr^2 + r^2\,(d\theta^2 + \sin^2\theta\, d\varphi^2)) \qquad (4.11)$$

4.22 – MOTION IN A STATIC FIELD WITH A SPHERICAL SYMMETRY

We determine the geodesics of the metric (We use physic units):

$$ds^2 = A\, c^2dt^2 - B\,(dr^2 + r^2\,(d\theta^2 + \sin^2\theta\, d\varphi^2)) \tag{4.12}$$

Where A and B are function of r with $A \approx 1$ and $B \approx 1$.
We consider the function L defined by:

$$L = A\, c^2dt^2/ds^2 - B\, dr^2/ds^2 - r^2\, B\,(d\theta^2/ds^2 + \sin^2\theta\, d\varphi^2/ds^2) \tag{4.13}$$

We note $'$ the derivation d /ds.

$$L = A\, c^2\, t'^{\,2} - B\, r'^{\,2} - r^2\, B\,(\theta'^{\,2} + \sin^2\theta\, \varphi'^{\,2}) \tag{4.14}$$

We write the Lagrange equations.

$$(\partial L/\partial q')' - \partial L/\partial q = 0 \tag{4.15}$$

with $q = t, \theta, \varphi$.

$$(A\, t')' = 0 \tag{4.16}$$

$$(B\, r^2\, \theta')' - r^2\, B \sin\theta \cos\theta\, \varphi'^{\,2} = 0 \tag{4.17}$$

$$(r^2\, B \sin^2\theta\, \varphi')' = 0 \tag{4.18}$$

(4.16) gives :

$$A\, t' = k/c$$

$$dt = k\, ds / Ac \tag{4.19}$$

where k = cst and k #1, (4.17) admits $\theta = \pi/2$ as particular solution, that corresponds to the motions around the star in the equatorial plane.

(4.17) gives then:

$$(r^2\, B\, \varphi')' = 0 \tag{4.20}$$

$$r^2\, B\, \varphi' = h/c$$

$$ds = r^2 c\, B\, d\varphi / h \tag{4.21}$$

where h = cst.

In (4.12) we replace dt by it value in (4.19) and with $\theta = \pi/2$, we obtains:

$$B\,dr^2 + r^2\,d\varphi^2 = (k^2/A - 1)\,ds^2 \qquad (4.22)$$

Now we substitute for ds with (4.21):

$$B\,(dr^2 + r^2\,d\varphi^2) = (k^2/A - 1)\,r^4 c^2 B^2\,d\varphi^2 / h^2 \qquad (4.23)$$

$$(d(1/r)/d\varphi)^2 = (k^2/A - 1)\,B\,c^2/h^2 - 1/r^2 \qquad (4.24)$$

We put $B = 1/A = e^{2mG/rc^2}$ and $u = 1/r$ in (4.24) and then we expand in series to the third order. We obtain:

$$(du/d\varphi)^2 = P(u) = c^2(k^2-1)/h^2 + 2G(2k^2-1)mu/h^2 - u^2 + 2G^2m^2(4k^2-1)\,u^2/c^2h^2 + 4G^3(8k^2-1)m^3u^3/(3c^4h^2) \qquad (4.25)$$

With this expression we can compute the advance of the perihelion of Mercury (see for example [10], pages 115 to 117), we obtain (with k = 1):

$$\delta\omega = 6G^2m^2\pi/c^2h^2 \qquad (4.26)$$

It is the value usually accepted.

4.3 – THE INTERIOR CASE

4.31– DETERMINATION OF THE FUNCTION f

We consider a material distribution without pressure (pure matter or dust) interacting with a gravitational field Φ by virtue of the theorem 4.1 its index F is:

$$F = f(\Phi) \qquad (4.27)$$

by virtue of the theorem (3.1) it exists a comoving coordinates system such as, if g is the metric tensor, $\gamma = F^2 g$, we have:

$$\gamma_{00} = F^2 g_{00} = f(\Phi)^2 g_{00} = 1 \qquad (4.28)$$

$$f(\Phi) = 1/\sqrt{(g_{00})} \qquad (4.29)$$

If on the analogy of (4.10) we want $g_{00} = e^{-2\Phi}$ then we must have:

$$\mathbf{f(\Phi) = e^{\Phi}} \qquad (4.30)$$

These considerations determine, in general, the function f. The equations of the theory become:

$$\mathbf{\Box\Phi = 4\pi\rho\, e^{\Phi}} \qquad \mathbf{(4.31)}$$

$$\mathbf{R_{\lambda\mu} - \tfrac{1}{2} R\, g_{\lambda\mu} = 8\pi e^{\Phi}((\rho + p)u_\lambda u_\mu - p\, g_{\lambda\mu}) - 2(\Phi_{,\lambda}\Phi_{,\mu} - \tfrac{1}{2} g_{\lambda\mu}\Phi_{,\lambda}\Phi^{,\lambda})} \qquad \mathbf{(4.32)}$$

It is important to observe that the quantities appearing in these equations, in particular ρ and p are measured in the Riemannian manifold (U, ds^2), a contrario the real values must be measured in (U, $d\sigma^2$); we have for example, with evident notations:

$$\rho_{real} = dm/dv_{real} = dm/(F^3 dv) = \rho/ F^3 \quad (4.33)$$

In the same way we have:

$$p_{real} = p/ F^3 \quad (4.34)$$

4.32– EQUATIONS WITH SPHERICAL SYMMETRY IN COMOVING COORDINATES SYSTEM

We utilize the metric (4.5), we have p = 0 and ρ ≠ 0.

$$ds^2 = e^{2a}dt^2 - e^{2b} (dr^2 + r^2 (d\theta^2 + \sin^2\theta\, d\varphi^2)) \quad (4.35)$$

a, b, ρ and the gravitational field Φ are some functions of r, the Einstein's equations (1.4) give:

$$(4b' + r\, b'^2 + 2r\, b'') /r\, e^{2b} = - 8\pi \rho\, e^{\Phi} + \Phi'^2 / e^{2b} \quad (4.36)$$

$$(2a' + 2b' + 2r\, a' b' + r\, b'^2) /r\, e^{2b} = - \Phi'^2 / e^{2b} \quad (4.37)$$

$$(a' + b' + r\, a'^2 + r\, a'' + r\, b'') /r\, e^{2b} = \Phi'^2 / e^{2b} \quad (4.38)$$

and the field equation for Φ:

$$\Box\Phi = - (2\Phi' + r\, a' \Phi' + r\, b' \Phi' + r\, \Phi'')/ r\, e^{2b} = 4\pi \rho\, e^{\Phi} \quad (4.39)$$

With (4.29) we obtain:

$$\underline{\mathbf{\Phi = -a}} \quad (4.40)$$

We replace Φ by -a in (4.36 to 39):

$$(4b' + r\, b'^2 + 2r\, b'') /r\, e^{2b} = - 8\pi \rho\, e^{-a} + a'^2 / e^{2b} \quad (4.41)$$

$$(2a' + 2b' + 2r\, a' b' + r\, b'^2) /r\, e^{2b} = - a'^2 / e^{2b} \quad (4.42)$$

$$(a' + b' + r\, a'^2 + r\, a'' + r\, b'') /r\, e^{2b} = a'^2 / e^{2b} \quad (4.43)$$

$$(2a' + r\, a'^2 + r\, a' b' + r\, a'')/ r\, e^{2b} = 4\pi \rho\, e^{-a} \quad (4.44)$$

In (4.44) we replace ρ by it value in (4.41):

$$(4b' + r\, b'^2 + 2r\, b'')/r\, e^{2b} = -2(2a' + r\, a'^2 + r\, a'\, b' + r\, a'')/r\, e^{2b} + a'^2 / e^{2b} \tag{4.45}$$

We simplify (4.45) then (4.42) and (4.43), we obtain:

$$4a' + 4b' + r\, a'^2 + r\, b'^2 + 2r\, a'\, b' + 2r\, a'' + 2r\, b'' = 0 \tag{4.46}$$

$$2a' + 2b' + r\, a'^2 + r\, b'^2 + 2r\, a'\, b' = 0 \tag{4.47}$$

$$a' + b' + r\, a'' + r\, b'' = 0 \tag{4.48}$$

In (4.46 to 48) we put $y = (a + b)$, we are:

$$4y' + r\, y'^2 + 2r\, y'' = 0 \tag{4.49}$$

$$y'(2 + r\, y') = 0 \tag{4.50}$$

$$y' + r\, y'' = 0 \tag{4.51}$$

The solutions are evident:

1) $y' = 0 \;\Leftrightarrow\; y = a + b = K = cst.$ (4.52)
Using a change of variable ($r \rightarrow \alpha r$) we can choose $K = 0$, we obtain:

$$\underline{\mathbf{b = -a = \Phi}} \tag{4.53}$$

The equation (4.44) becomes:

$$(2\Phi' + r\, \Phi'')/r = -4\pi\, \rho\, e^{3\Phi} \tag{4.54}$$

This equation permits, knowing ρ, the determination of the field Φ, this situation is the same one as in classic mechanics, it is not the case in RG. For the metrics we have:

$$ds^2 = e^{-2\Phi}dt^2 - e^{2\Phi}(dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)) \tag{4.55}$$

$$d\sigma^2 = e^{2\Phi}ds^2 = dt^2 - e^{4\Phi}(dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)) \tag{4.56}$$

The metric $d\sigma^2$ is the frame of the physics and all the measures must be done with its.

2) $2 + r\, y' = 0 \Leftrightarrow a' + b' = -2/r \Leftrightarrow b = -a - \ln r^2 \Leftrightarrow b = \Phi - \ln r^2 \Leftrightarrow e^b = e^{\Phi}/r^2$. We have:

$$ds^2 = e^{-2\Phi}dt^2 - e^{2\Phi}(dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2))/r^4 \tag{4.57}$$

$$ds^2 = e^{-2\Phi}dt^2 - e^{2\Phi}(dr^2/r^4 + (d\theta^2 + \sin^2\theta\, d\varphi^2)/r^2) \tag{4.58}$$

We $u = 1/r$ and we obtain:

$$ds^2 = e^{-2\Phi}dt^2 - e^{2\Phi}(du^2 + u^2(d\theta^2 + \sin^2\theta\, d\varphi^2)) \tag{4.59}$$

We return to the first case.

5– APPLICATIONS

For example we remember Einstein, in the year 1917, wanted to build a static hyper-spherical universe filled up pure matter, and with this intention, he has introduced the cosmological constant. In our theory that constant is not necessary. The metric of the static hyper-spherical universe is:

$$ds^2 = dt^2 - (dr^2 + r^2 (d\theta^2 + \sin^2\theta\, d\varphi^2))/ (1+r^2/4a^2)^2 \tag{5.1}$$

where a is a constant strictly positive. The comparison between (5.1) and (4.56) gives:

$$\Phi = -\ln(1 + r^2/4a^2)/2 \tag{5.2}$$

Then the equation (4.54) gives:

$$\rho = - e^{-3\Phi} (2\Phi' + r\, \Phi'')/4\pi r \tag{5.3}$$

The relations (4.30) and (4.33) give:

$$\rho_{real} = \rho\, e^{-3\Phi} = - e^{-6\Phi} (2\Phi' + r\, \Phi'')/4\pi r = (4a^2 + r^2)(12a^2 + r^2)/ 256\pi\, a^6 \tag{5.4}$$

We can compute the mass of that universe, it is infinite.
Now that universe has only a historic interest but one never knows.

6 – CONCLUSION

The equality of the inertial mass and the gravitational mass do not imply necessarily the weak principle of equivalence. The theory presented in this paper makes the distinction between the gravitational field and the inertial field. It gives the correct value for the advance of the perihelion of Mercury but on the over hand it presents several interesting and innovative points.

Firstly the analogue of the Schwarzschild solution does not present a singularity except the origin.

Secondly it is possible to build a stable mass of matter as large as one wants.

These last considerations show the possibility to re-examine the theory of the black holes.

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